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Effective Field Equations of the Quantum Gravitational Back-Reaction on Inflation

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Quantum gravitational back-reaction offers the potential of simultaneously resolving the problem of the cosmological constant and providing a natural model of inflation in which scalars play no special role. In this model inflation begins because the cosmological constant is not unnaturally small. It ends through the accumulated gravitational interaction between virtual gravitons which are ripped apart by the inflationary expansion. Although perturbative techniques can be used to study the effect as long as it remains weak, they break down when back-reaction begins to exert an appreciable effect on the expansion rate. In this talk I argue that the end of inflation is sudden and that there is actually an overshoot into deflation. (This incidentally provides a very efficient mechanism for reheating.) The subsequent evolution can be understood in terms of a competition between the opening of the past light cone and the formation of a thermal barrier to the persistence of correlations from during the period of inflation.

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1 Introduction

Quantum gravitational back-reaction offers an attractive model of cosmology which also resolves the problem of the cosmological constant. The idea is that there is no fine tuning of the cosmological constant, Λ, or of scalar potentials [1]. In fact there need not be any scalars. Inflation begins in the early universe because Λ is positive and not unnaturally small. Inflation eventually ends due to the accumulation of gravitational attraction between long wavelength virtual gravitons which are pulled apart by the rapid expansion of spacetime. Inflation persists for many e-foldings because gravity is a weak interaction, even at typical inflationary scales, and it requires an enormous accumulation of gravitational potential to overcome this. Because the model has only a single free parameter — $G\Lambda$, where G is Newton's constant — it can be used to make unique and testable predictions. For example, when Λ is adjusted (to a mass scale of about $.72 \times 10^{16}$ GeV) so as to make the COBE 4-year RMS quadrupole[2] come out right, the model gives the following predictions for the parameters which describe primordial anisotropies in the cosmic microwave background [3]:

$$r \approx .0017 \qquad , \qquad n_s \approx .97 \qquad , \qquad n_t \approx -.00028 \; . \tag{1}$$

The mechanism through which long wavelength gravitons are pulled out of the vacuum in an inflating universe is known as *superadiabatic amplification* [4]. It can be understood as the consequence of three well-known results. The first is that the energy-time uncertainty principle implies the continual emergence of all types of virtual quanta from empty space. The second fact is that inflation provides a geometry for which Zeno's argument against the possibility of motion is sometimes valid. Even light cannot pass between co-moving observers which are separated by more than a Hubble length, H^{-1}, where $H \equiv \sqrt{\Lambda/3}$. By the time the light has traveled halfway, the intervening space has expanded so much that the distance remaining is actually greater than at the beginning. It follows that virtual pairs of wavelength comparable to or greater than H^{-1} cannot recombine. The final fact is that the graviton's masslessness and lack of conformal invariance imply an appreciable rate for the emission of virtual quanta of Hubble wavelength. It is significant that the analogous process for light, minimally coupled scalars is thought to be the origin of the observed anisotropies in the cosmic microwave background [5, 6]. This is solid physics.

Particles which are massive at or above the scale of inflation do not experience superadiabatic amplification because the amplitude for producing virtual quanta of Hubble wavelength is so small. Nor is there any superadiabatic amplification of particles which are conformally invariant on the classical level. Their quanta cannot locally sense the expansion of spacetime because the geometry of an inflating universe tends rapidly towards homogeneity and isotropy, and hence to conformal flatness. This means that the graviton is almost unique. If a minimally coupled scalar can somehow avoid acquiring mass at or above the scale of inflation it can also experience superadiabatic amplification. All other

particles are either massive or else they possess classical conformal invariance. If one was searching for an arena in which quantum gravitational effects might be significant, the long wavelength sector of inflationary cosmology would be a natural choice.

The infrared character of the physical mechanism means that it can be studied reliably using perturbative quantum general relativity, without requiring a fundamental theory of quantum gravity. Loops of massless particles give rise to nonlocal and ultraviolet finite terms which cannot be subsumed into local counterterms and which are not affected by changes in the short wavelength sector. Infrared phenomena can therefore be studied using the low energy effective theory. This is why Bloch and Nordsieck [7] were able to resolve the infrared problem in QED before the theory's renormalizability was suspected. It is also why Weinberg [8] was able to give a similar resolution for the infrared problem of quantum gravity with zero cosmological constant. And it is why Feinberg and Sucher [9] were able to compute the long range force induced by neutrino exchange using Fermi theory. Extensive work along these same lines has been done recently on quantum gravity with zero cosmological constant by Donoghue [10].

Nick Tsamis and I have done a two loop computation of the pure quantum gravitational back-reaction on inflation [11]. The expectation value of the gauge-fixed metric was computed on the manifold $T^3 \times R$ and in the presence of a state which is free graviton vacuum at $t = 0$:

$$\Big\langle \Omega \Big| g_{\mu\nu}(t, \vec{x}) dx^\mu dx^\nu \Big| \Omega \Big\rangle = -dt^2 + e^{2b(t)} d\vec{x} \cdot d\vec{x} \; . \tag{2}$$

Treating this expectation value as if it were the classical invariant element, one forms the effective Hubble constant,

$$\dot{b}(t) = H \left\{ 1 - \left(\frac{G\Lambda}{3\pi}\right)^2 \left(\frac{1}{6}(Ht)^2 + O(Ht)\right) + O(G^3) \right\} \; . \tag{3}$$

The correct interpretation of this result is that quantum gravitational back-reaction slows inflation as long as perturbation theory remains valid, and that the effect must eventually grow nonperturbatively strong.

The great unsolved problem is how to analytically treat the period beyond the end of inflation when perturbation theory is no longer valid. That progress is possible derives from the peculiar manner in which slowing becomes nonperturbative. This is not a strongly coupled theory like QCD; the dimensionless coupling constant is $G\Lambda \approx 5 \times 10^{-11}$. It is not that any particular particle creation event matters much but rather that an enormous number of them add coherently over the ever-increasing volume of the past light cone. Those factors of Ht in (3) derive from the fact that there are two vertices being integrated over the past light cone, whose invariant volume during inflation is,

$$V(t) = \int_0^t dt' e^{3b(t')} \frac{4}{3}\pi \left[\int_{t'}^t dt'' e^{-b(t'')}\right]^3 \approx \frac{4}{3}\pi H^{-4} Ht \; . \tag{4}$$

It is therefore reasonable to expect that the incremental screening from each volume element of the past light cone is always minuscule. These increments can be computed perturbatively if only we can learn how to work in the changing background.

My purpose here is to outline the physical principles and techniques by which Dr. Tsamis and myself are attempting to gain analytic control over the nonperturbative regime. Section 2 describes the analog scalar model with which we test ideas. Section 3 is devoted to a new infrared expansion of the mode functions which allows us to obtain the propagators as the background changes. Section 4 argues that screening becomes so strong at the end of inflation that there is no alternative to a brief period of deflation. This leads to the rapid formation of a hot, dense plasma of ultraviolet particles which we call, the *thermal barrier*. The thermal barrier scatters the infrared virtual quanta which need to carry quantum correlations into the future from the period of inflationary particle production in order to keep the large bare cosmological constant screened. When the barrier becomes too effective, deflation ends and the universe begins to expand again. Section 5 discusses the balance between the thermal barrier and the growth of the inflationary past light cone which we believe governs this subsequent phase of expansion. Since we have not yet proven the viability of this scheme I can offer no definite conclusions. I close instead by mentioning the potential payoff if these ideas lead to a workable model.

2 The scalar analog model

As discussed in the previous section, the effect we seek to study has two essential features. The first is that infrared virtual particles are continually being ripped out of the vacuum and pulled apart by the inflationary expansion of spacetime. The second crucial feature is that these particles attract one another through a weak long range force which gradually accumulates as more and more particles are created. The particles we believe were actually responsible for stopping inflation are gravitons, and the long range force through which they did it was gravitation. However, this is not a simple theoretical setting in which to work. It took over a year of labor to obtain the two loop result (3) — even with computer symbolic manipulation programs [11]. Before attempting to test speculative ideas in quantum general relativity it is natural to wonder if there is not a simpler theory which manifests the same effect.

As also explained in the previous section, the prerequisites for inflationary particle production are masslessness on the scale of inflation and the absence of classical conformal invariance. Massless, minimally coupled scalars have these properties — and the lowest order back-reaction from self-interacting scalars can be worked out on a blackboard in about 15 minutes [12]. Of course it is not natural for scalars to possess attractive self-interactions and still remain massless on the scale of inflation. But we do not need a *realistic* model — that is already provided by gravitation. What we seek is rather a *simple* model that can be tuned to show the same physics, however contrived and unnatural this tuning may be.

A massless, minimally coupled scalar with a nonderivative self-interaction seems to do

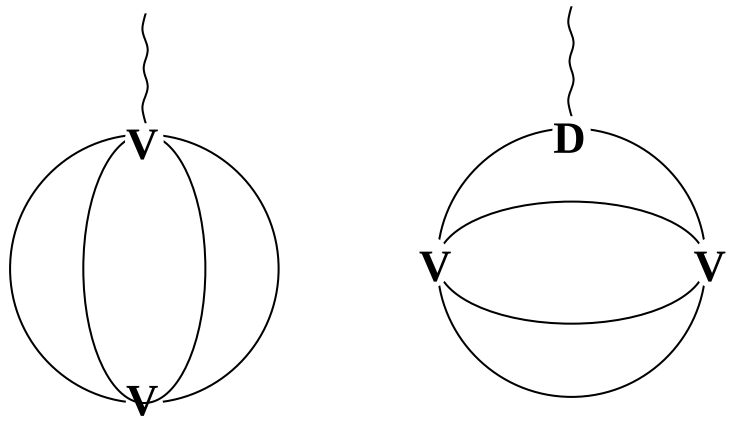

Figure 1: Contributions to the scalar stress-energy tensor at order λ^2 *with* covariant normal ordering. V denotes the 4-point vertex and D represents the derivative vertex.

what we want,

$$\mathcal{L} = -\frac{1}{2} : \partial_\mu \phi \partial_\nu \phi : g^{\mu\nu} \sqrt{-g} - \frac{1}{4!} \lambda : \phi^4 : \sqrt{-g} + \Delta\mathcal{L} \; . \tag{5}$$

The colons stand for a covariant normal-ordering prescription whose effect is to subtract off contributions from coincident propagators [13]. The counterterms $\Delta\mathcal{L}$ are used to enforce masslessness and the vanishing of the stress-energy tensor at the initial time. We are not quantizing gravity. The metric is a non-dynamical background which we take to be locally de Sitter in conformal coordinates,

$$g_{\mu\nu}(\eta, \vec{x}) = \Omega^2(\eta)\eta_{\mu\nu} \qquad , \qquad \Omega(\eta) = -\frac{1}{H\eta} = e^{Ht} \; . \tag{6}$$

(Recall that $\Lambda = 3H^2$.) To regulate the infrared problem on the initial value surface we work on the manifold $T^3 \times R$, with the spatial coordinates in the finite range, $-H^{-1}/2 < x^i \leq H^{-1}/2$. We release the state in Bunch-Davies vacuum at $t = 0$, corresponding to conformal time $\eta = -H^{-1}$. Note that the infinite future corresponds to $\eta \to 0^-$, so the possible variation of conformal coordinates in either space or time is at most $\Delta x = \Delta\eta = H^{-1}$.

The stress-energy tensor consists of the potential term plus derivatives which are subdominant [12],

$$T_{\mu\nu}(x) = -g_{\mu\nu}(x) \frac{\lambda}{4!} : \phi^4(x) : + \text{subdominant} \; . \tag{7}$$

Because there are no coincident propagators the lowest contribution to its expectation value comes at order λ^2 from the two diagrams in Fig. 1 [12]. (Of course there is a

cosmological counterterm to absorb the ultraviolet divergence.) Further, the derivatives on the top vertex of the right hand diagram render its contribution subdominant to the left hand diagram in powers of Ht. So the dominant contribution to the expectation value of the stress-energy tensor is $-g_{\mu\nu}(x)$ times,

$$\frac{\lambda}{4!}\Big\langle \Omega \Big| : \phi^4(x) : \Big| \Omega \Big\rangle = \\ \frac{-i\lambda^2}{4!}\int_{t'>0} d^4x' \Omega^4(\eta') \Big\{ [i\Delta_{++}(x;x')]^4 - [i\Delta_{+-}(x;x')]^4 \Big\} + O(\lambda^3) \; . \tag{8}$$

The difference of $++$ and $+-$ propagators,*

$$i\Delta_{++}(x;x') \equiv \\ \frac{1}{4\pi^2}\frac{\Omega^{-1}(\eta)\Omega^{-1}(\eta')}{\Delta x^2 - (|\Delta\eta| - i\epsilon)^2} - \frac{H^2}{8\pi^2}\ln\Big[H^2\Big(\Delta x^2 - (|\Delta\eta| - i\epsilon)^2\Big)\Big] \; , \tag{9}$$

$$i\Delta_{+-}(x;x') \equiv \\ \frac{1}{4\pi^2}\frac{\Omega^{-1}(\eta)\Omega^{-1}(\eta')}{\Delta x^2 - (\Delta\eta + i\epsilon)^2} - \frac{H^2}{8\pi^2}\ln\Big[H^2\Big(\Delta x^2 - (\Delta\eta + i\epsilon)^2\Big)\Big] \; , \tag{10}$$

comes from using the Schwinger-Keldysh formalism [14, 15] to compute an expectation value rather than an in-out amplitude. This form ensures that the result is real and that it depends only upon points x'^μ in the past light cone of the observation point x^μ. The lower limit of temporal integration at $\eta' = -H^{-1}$ (that is, $t' = 0$) derives from the fact that we release the state in free Bunch-Davies vacuum at this instant.

Although (8) was computed in ref. [12] we will go over it in detail. Since only the logarithm term of the propagator breaks conformal invariance it is perhaps not surprising that the dominant secular effect comes from taking this term in each of the four propagators. This contribution is completely ultraviolet finite, and its evaluation is straightforward if one goes after only the largest number of temporal logarithms,

$$\frac{-i\lambda^2}{4!}\left(\frac{-H^2}{8\pi^2}\right)^4 \int_{-H^{-1}}^{\eta} d\eta' \left(\frac{-1}{H\eta'}\right)^4 4\pi \int_0^\infty dr r^2 \\ \times \Big\{ \ln^4\Big[H^2\Big(r^2 - (\Delta\eta - i\epsilon)^2\Big)\Big] - \ln^4\Big[H^2\Big(r^2 - (\Delta\eta + i\epsilon)^2\Big)\Big]\Big\}$$

$$\rightarrow \frac{-i\lambda^2 H^4}{2^{13}3^1\pi^7}\int_{-H^{-1}}^{\eta} d\eta' \frac{1}{\eta'^4}\int_0^{\Delta\eta} dr r^2 \, 8\pi i \, \ln^3\Big[H^2(\Delta\eta^2 - r^2)\Big] \; , \tag{11}$$

$$= \frac{\lambda^2 H^4}{2^{10}3^1\pi^6}\int_{-H^{-1}}^{\eta} d\eta' \frac{\Delta\eta^3}{\eta'^4}\int_0^1 dx x^2 \Big[2\ln(H\Delta\eta) + \ln(1-x^2)\Big]^3 \; , \tag{12}$$

$$\rightarrow \frac{\lambda^2 H^4}{2^7 3^2\pi^6}\int_{-H^{-1}}^{\eta} d\eta' \frac{\Delta\eta^3}{\eta'^4}\ln^3(H\Delta\eta) \; . \tag{13}$$

*The conformal coordinate separations in these formulae are, $\Delta x \equiv \|\vec{x} - \vec{x}'\|$ and $\Delta\eta \equiv \eta - \eta'$.

For large Ht the biggest effect comes from the term with the most factors of $\ln(-H\eta) = -Ht$. That the integrand contributes three such factors follows from the expansion,

$$\ln(H\Delta\eta) = \ln(-H\eta') - \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{\eta}{\eta'}\right)^n . \tag{14}$$

An additional factor comes from performing the integration up against the final term in the expansion of the ratio,

$$\frac{\Delta\eta^3}{\eta'^4} = \frac{\eta^3}{\eta'^4} - 3\frac{\eta^2}{\eta'^3} + 3\frac{\eta}{\eta'^2} - \frac{1}{\eta'} . \tag{15}$$

The final result is therefore,

$$\frac{\lambda}{4!} \Big\langle \Omega \Big| : \phi^4(x) : \Big| \Omega \Big\rangle = -\frac{\lambda^2 H^4}{2^9 3^2 \pi^6} \Big\{ (Ht)^4 + O(H^3 t^3) \Big\} + O(\lambda^3) . \tag{16}$$

Two points deserve comment. First, there is nothing paradoxical about the negative sign of the $(Ht)^4$ contribution to the expectation value of a positive operator. The actual result is dominated by a positive ultraviolet divergent constant. It is only after the cosmological counterterm is used to subtract this divergence that the ultraviolet finite factor of $(Ht)^4$ dominates the late time behavior of the scalar stress-energy tensor at order λ^2.

Our second comment is that the negative sign of the $(Ht)^4$ term has a simple physical interpretation. As the inflationary expansion rips more and more scalars out of the vacuum their attractive self-interaction acts to pull them back together. The resulting expansion rate is [12],

$$\dot{b}(t) = H \left\{ 1 - \frac{\lambda^2 G \Lambda}{2^7 3^4 \pi^5} \Big[(Ht)^4 + O(H^3 t^3) \Big] + O(\lambda^3, G^2) \right\} . \tag{17}$$

This is the direct analog of the graviton effect we have been seeking. We turn now to the problem of computing the left hand diagram of Fig. 1 in the presence of the back-reacted geometry.

3 A new infrared expansion

It is straightforward to work out the interaction vertices in any background. The great obstacle to computing in different backgrounds is finding the propagator. We need to do this in a homogeneous, isotropic and spatially flat background,

$$g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + e^{2b(t)} d\vec{x} \cdot d\vec{x} = \Omega^2(\eta) \Big(-d\eta^2 + d\vec{x} \cdot d\vec{x} \Big) , \tag{18}$$

which began de Sitter inflation at $\eta = -1/H$ in free Bunch-Davies vacuum. The problem here is solving for $u(\eta, k)$, the mode function for a scalar of co-moving wave number k. It obeys the equation,

$$u''(\eta, k) + 2\frac{\Omega'}{\Omega} u'(\eta, k) + k^2 u(\eta, k) = 0 . \tag{19}$$

Canonical quantization and the initial Bunch-Davies vacuum fixes the initial conditions (up to a phase) as,

$$u\Big(-H^{-1},k\Big) = \frac{iH}{\sqrt{2k^3}} e^{ik/H} \Big\{ 1 - i\frac{k}{H} \Big\} \quad , \quad u'\Big(-H^{-1},k\Big) = \frac{iH}{\sqrt{2k^3}} e^{ik/H} \Big\{ -\frac{k^2}{H} \Big\} \; . \tag{20}$$

If we could solve (19) for arbitrary conformal factor $\Omega(\eta)$ the $++$ and $+-$ propagators would be,

$$i\Delta_{++}(x;x') = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\Delta\vec{x}} \Big\{ \theta(\Delta\eta) u(\eta,k) u^*(\eta',k) + \theta(-\Delta\eta) u^*(\eta,k) u(\eta',k) \Big\} \; , \tag{21}$$

$$i\Delta_{+-}(x;x') = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\Delta\vec{x}} u^*(\eta,k) u(\eta',k) \Big\} \; . \tag{22}$$

Here we define the conformal coordinate separations as, $\Delta\vec{x} \equiv \vec{x} - \vec{x}'$ and $\Delta\eta \equiv \eta - \eta'$. A closely related and often useful quantity is the retarded Green's function,

$$G_{\rm ret}(x;x') = \theta(\Delta\eta) \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\Delta\vec{x}} {\rm Im}\Big\{ u(\eta,k) u^*(\eta',k) \Big\} \; . \tag{23}$$

Since the physical effect we are studying is infrared we actually require only an expansion which is valid for long wavelengths. What this means changes with time owing to the expansion of spacetime. During inflation we distinguish between modes which infrared (IR) and ultraviolet (UV) by comparing their co-moving wavenumbers thusly,

$$H < {\rm IR} < \frac{\Omega'(\eta)}{\Omega(\eta)} < {\rm UV} \; . \tag{24}$$

Suppose inflation ends at conformal time η_1, and that subsequent evolution consists either of deflation or subluminal expansion. Then some of the IR modes which experienced horizon crossing during inflation re-enter the horizon. We designate these modes as thermal (TH) and distinguish them thusly,

$$H < {\rm IR} < \left| \frac{\Omega'(\eta)}{\Omega(\eta)} \right| < {\rm TH} < \frac{\Omega'(\eta_1)}{\Omega(\eta_1)} < {\rm UV} \; . \tag{25}$$

Since we need an infrared expansion the straightforward approach would be to solve (19) perturbatively, regarding the k^2 term as the small parameter. This turns out not to give a satisfactory expansion for a number of reasons [16, 17]. One problem is that the initial conditions involve fractional, and even inverse powers of the small parameter. Another problem is that although causality requires the full retarded Green's function (23) to vanish for $\Delta x > \Delta\eta$, this feature is not manifest at any finite order of the k^2

expansion. A better strategy seems to be first factoring out a temporal exponential and a multiplicative normalization,

$$u(\eta, k) \equiv \frac{iH}{\sqrt{2k^3}} e^{-ik\eta} q(\eta, k) \; . \tag{26}$$

We then perturb, regarding ik as the small parameter,

$$q''(\eta, k) + 2\frac{\Omega'}{\Omega} q'(\eta, k) = 2ik \left\{ q'(\eta, k) + \frac{\Omega'}{\Omega} q(\eta, k) \right\} \; . \tag{27}$$

The initial conditions (20) become,

$$q\left(-H^{-1}, k\right) = 1 - i\frac{k}{H} \qquad , \qquad q'\left(-H^{-1}, k\right) = ik \; . \tag{28}$$

Making the substitution,

$$q(\eta, k) = \sum_{n=0}^{\infty} q_n(\eta)(ik)^n \; , \tag{29}$$

we easily find the first two terms,

$$q_0(\eta) = 1 \qquad , \qquad q_1(\eta) = \eta \; . \tag{30}$$

The next order term is,

$$q_2(\eta) \equiv \frac{1}{2}\Big(\eta^2 - H^{-2}\Big) + H^{-1} \int_{-1/H}^{\eta} d\eta' \Omega^{-2}(\eta') + \int_{-1/H}^{\eta} d\eta' \Omega^{-2}(\eta') \int_{-1/H}^{\eta'} d\eta'' \Omega^2(\eta'') \; , \tag{31}$$

and the higher terms are obtained through the recursion relation,

$$q_n(\eta) \equiv \int_{-1/H}^{\eta} d\eta' q_{n-1}(\eta') + \int_{-1/H}^{\eta} d\eta' \Omega^{-2}(\eta') \int_{-1/H}^{\eta'} d\eta'' \Omega^2(\eta'') q'_{n-1}(\eta'') \; . \tag{32}$$

This expansion avoids the problems associated with the long wavelength limit [16, 17]. It has the potential for extending to the ultraviolet regime by virtue of the oscillatory behavior of the factor of $e^{-ik\eta}$. The first two terms are also exact for pure de Sitter, and the higher terms can be evaluated in the slow roll expansion. Finally, applying the expansion to the mode sum for the retarded Green's function results in a series of higher and higher derivatives of the past light cone theta function $\theta(\Delta\eta - \Delta x)$,

$$G_{\rm ret}(x; x') = -\theta(\Delta\eta) \frac{H^2}{4\pi} \left\{ \theta(\Delta\eta - \Delta x) + \frac{\eta\eta'}{\Delta x} \delta(\Delta\eta - \Delta x) + \dots \right\} \; . \tag{33}$$

This certainly seems to be organized as an infrared expansion should be, with the large volume effects at lowest order.

4 Post-inflationary deflation

The cosmological constant is screened by the coherent superposition of the attractive long range potentials produced by virtual particles which are ripped apart during inflation. Although any one particle creation event makes only a small contribution, the coherent superposition of events from the entire past light cone can be enormous. Two things happen when the expansion rate begins to slow appreciably: the volume rate of particle production drops and the growth of the past light cone increases. The first can not have much immediate effect on screening because most of the past light cone at the end of inflation derives from periods when the particle production rate was still high. It is the second effect which dominates. Slower expansion means that more and more invariant volume from the period of inflationary particle production becomes visible. Further, most of the propagation of this effect occurs through the period of rapid expansion when the de Sitter approximation is still valid. Not only does screening continue to grow after the end of inflation; its rate of growth actually *increases*.

The situation I have just described is a classic runaway instability. The lower the expansion rate drops the faster the attractive self-interaction accumulates. There seems to be no alternative to a rapid decay into deflation. To gain a rough understanding of what happens next let us solve for the mode functions under the assumption that pure de Sitter inflation goes over to pure de Sitter deflation at conformal time η_1,

$$\Omega(\eta) = \begin{cases} -1/H\eta, & -H^{-1} \leq \eta \leq \eta_1; \\ 1/H(\eta - 2\eta_1), & \eta_1 \leq \eta \ . \end{cases} \tag{34}$$

The inflationary mode function was given in the previous section,

$$u(\eta, k) = \frac{iH}{\sqrt{2k^3}}(1 + ik\eta)e^{-ik\eta} \qquad \forall -H^{-1} \leq \eta \leq \eta_1 \ . \tag{35}$$

Since the conformal factor is continuous at $\eta = \eta_1$ the mode function is also continuous there, as is its first derivative. The deflationary mode function can be most simply expressed in terms of another function $v(\eta, k)$,

$$u(\eta, k) = -v(\eta, k) + \frac{ie^{-ik\eta_1}}{k\eta_1}\Big[e^{ik\eta_1} v(\eta, k) + e^{-ik\eta_1} v^*(\eta, k)\Big] \qquad \forall\ \eta_1 \leq \eta \ , \tag{36}$$

$$v(\eta, k) = \frac{iH}{\sqrt{2k^3}}\Big(1 + ik(\eta - 2\eta_1)\Big)e^{-ik\eta} \ . \tag{37}$$

Note the factor of $1/\eta_1$ on the second term in (36). Since screening requires an enormous number of e-foldings to become effective, we can assume that η_1 is an infinitesimal negative number. It follows that the second term of (36) rapidly becomes huge. Since this appears to make screening *even stronger* there does not seem to be any restoring force against runaway deflation!

Appearances are deceiving. The growth in the second term of (36) has a physical origin which becomes clear upon simplification,

$$u(\eta,k) = -v(\eta,k) + i e^{-ik\eta_1} \sqrt{\frac{2}{H}} \left(\frac{H}{k}\right)^{\frac{3}{2}} \frac{\Omega_1}{\Omega(\eta)} \left\{ \cos\left[k(\eta-\eta_1)\right] - \frac{\sin[k(\eta-\eta_1)]}{k(\eta-2\eta_1)} \right\} . \quad (38)$$

Now specialize the classification of co-moving wave numbers to the deflationary geometry,

$$H < \mathrm{IR} < \frac{1}{\eta - 2\eta_1} < \mathrm{TH} < \frac{-1}{\eta_1} < \mathrm{UV} . \quad (39)$$

After any significant amount of deflation we have $\eta \gg -\eta_1$. We can therefore conclude that most infrared modes obey $k\eta \ll 1$ and hence,

$$\cos\left[k(\eta-\eta_1)\right] - \frac{\sin[k(\eta-\eta_1)]}{k(\eta-2\eta_1)} \approx -\frac{1}{3}(k\eta)^2 . \quad (40)$$

The second term is therefore not large for modes which are still infrared. However, most of the infrared modes which have re-entered the horizon — which we call "thermal" — obey $1 \ll k\eta$. For them the cosine term is of order one whereas the sine is minuscule. The result is that the second term of (38) grows exponentially like the inverse scale factor,

$$\frac{\Omega_1}{\Omega(\eta)} = e^{H(t-t_1)} . \quad (41)$$

What we are seeing in the second term is nothing more than the blue shift due to deflation. The thermal modes have been populated by the particle production that went on during inflation, but after re-entering the horizon they obey the equation of state of radiation. As the universe deflates they form a hot, dense plasma.

It is not correct to treat this second term as part of the mode function for a quantum field. It is rather a stochastic background whose precise value is random but definite in the sense advocated by Linde [18, 17]. To understand this let us form the Fourier transform of the free field by multiplying the mode functions by canonically normalized creation and annihilation operators,

$$\widetilde{\phi}(\eta,\vec{k}) = u(\eta,k) a(\vec{k}) + u^*(\eta,k) a^\dagger(\vec{k}) . \quad (42)$$

This field commutes canonically with Ω^2 times it conformal time derivative. Suppose we decompose the field as we have the mode function (38). This results in two terms,

$$\begin{aligned} \widetilde{\phi}_{\rm qm}(\eta,\vec{k}) &\equiv -v(\eta,k) a(\vec{k}) - v^*(\eta,k) a^\dagger(\vec{k}) , & (43) \\ \widetilde{\phi}_{\rm big}(\eta,\vec{k}) &\equiv \sqrt{\frac{2}{H}} \left(\frac{H}{k}\right)^{\frac{3}{2}} \frac{\Omega_1}{\Omega(\eta)} \left\{ \cos\left[k(\eta-\eta_1)\right] - \frac{\sin[k(\eta-\eta_1)]}{k(\eta-2\eta_1)} \right\} \\ &\qquad \times \left[i e^{-ik\eta_1} a(\vec{k}) - i e^{ik\eta_1} a^\dagger(\vec{k}) \right] . & (44) \end{aligned}$$

The first of these, $\widetilde{\phi}_{\rm qm}$ obeys the same commutation relations as the full field. However, $\widetilde{\phi}_{\rm big}$ *commutes* with its conformal time derivative. It is still an operator, in that it has the potential for taking any value, but it behaves like a classical field once this value is chosen.

One must subsume $\widetilde{\phi}_{\rm big}(\eta, \vec{k})$ into the background. As we saw, it is only significant for the thermal modes. These terms should be treated as a classical gas. Doing this permits us to keep using perturbation theory. Far more important, it also provides the physical mechanism by which deflation is halted. For note that screening is maintained by the propagation into the future of quantum correlations from the period of inflationary particle production. This is accomplished by the infrared modes. But they will be scattered by the thermal particles, and such a collision is overwhelmingly likely to bump them up into the high momentum regime. We can therefore view the thermal modes as forming a partial barrier to the forward propagation of the infrared modes. As the universe deflates the barrier becomes thicker and thicker. Eventually it overcomes the continued growth of the past light cone, at which point the bare cosmological constant begins to reassert itself and expansion resumes.

Computing the scattering rate, $\Gamma(\eta, k)$, is a trivial exercise in perturbation theory and has already been done under the assumption that the previous expansion for $u(\eta, k)$ is valid in the absence of the barrier. The depletion effect can be roughly accounted for by degrading the infrared mode functions by the following replacement,

$$u(\eta, k) \longrightarrow \exp\left[-\frac{1}{2}\int^{\eta} d\eta' \Gamma(\eta', k)\right] \times u(\eta, k) \; . \qquad (45)$$

Simple phase space arguments imply that $\Gamma(\eta, k)$ is proportional to $1/k$. The wonderful feature of this result is that the mode sum for the propagator can be evaluated analytically using the integral,

$$\int_0^{\infty} dk k^{\nu - 1} \exp\left[-\frac{\beta}{k} - \gamma k\right] = 2 \left(\frac{\beta}{\gamma}\right)^{\frac{\nu}{2}} K_{\nu}\left(2\sqrt{\beta\gamma}\right) \qquad {\rm Re}(\beta) > 0 \; , \; {\rm Re}(\gamma) > 0 \; . \qquad (46)$$

So it should be possible to evaluate the strength of the quantum effect as the geometry back-reacts.

5 Subsequent evolution

Post inflationary evolution is controlled by the balance between the degradation of the screening effect by the thermal barrier, and the fact that more and more of the inflationary past light cone is visible at later and later times. The key to stability is the thermal barrier. If the expansion rate becomes too high the density of particles in the barrier decreases, which reduces the new scatters of infrared quanta and therefore makes screening more effective. If the expansion rate slows too much the particle density falls off slower, leading

to more new scatters and a decline in screening. Note that the growth of the past light cone has a destabilizing effect since it grows more slowly when the expansion rate is higher. What gives rise to stability is that the thermal degradation effect acts more strongly and more quickly.

The growth of the past light cone is a purely geometrical effect which can be usefully approximated as follows. Suppose that inflation ends suddenly at conformal time η_1 (co-moving time t_1). Then the invariant volume of the inflationary past light cone which is visible at some later conformal time η (co-moving time t) is,

$$V(t) = \frac{4}{3}\pi \int_{-1/H}^{\eta_1} d\eta' \Omega^4(\eta')(\eta - \eta')^3 \approx \frac{4}{3}\pi \Big\{ H t_1 + I^3(t) \Big\} \ , \tag{47}$$

where the function $I(t)$ is,

$$I(t) \equiv H e^{b(t_1)} \int_{t_1}^{t} dt' e^{-b(t')} \ . \tag{48}$$

Because the universe deflates immediately after inflation $b(t_1) > b(t)$, so $I(t)$ rapidly comes to dominate the constant term in (47).

The nature of the thermal barrier will of course depend upon the available matter quanta into which energy flows through thermalization. Combining the various effects should result in a nonlocal equation for the logarithmic scale factor $b(t)$ which can be evolved numerically. Questions to be answered include:

1. What reheat temperature is reached?
2. What is the asymptotic form of $b(t)$?
3. What is the impact on structure formation?
4. How does the model respond to late time phase transitions?
5. Does the model possess a late time phase of acceleration such as seems to be occurring now [19, 20]?

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